Quiz 3 solutions

Solution 2: False Let $g(x) = -\frac{1}{x^2}$ and $f(x) = -\frac{1}{x}$. Notice that $f(x) \le g(x)$ and $\int_a^\infty g(x)\,dx$ converges. However, $\int_a^\infty f(x)\,dx$ is not convergent. **Note:** The statement was incorrectly set as true in WeBWorK. Full mark was given to all students.

Solution 3: False Consider two rectangles with base on the x-axis of different dimensions but same area.

Solution 4: The integral

$$I = \int_1^\infty \frac{x^{82}}{x^{np+1}}\,dx = \int_1^\infty \frac{1}{x^{np+1-82}}\,dx$$

converges if $np + 1 - 82 > 1$ and diverges if $np + 1 - 82 \le 1$. So, the statement integral diverges if $p > \frac{41}{n}$ is false. **Note:** There is an error in WeBWorK version of this question. Full mark was given to all students.

Solution 5: The general partial fraction decomposition of

$$f(x) = \frac{x^3 + 1}{(x^2 - 16)(x^2 + 16)}$$

is

$$\frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x+4} + \frac{D}{(x+4)^2} + \frac{Ex + F}{x^2 + 16}.$$

Solution 6: WeBWorK solution of the question: The average value of the function $v(x) = 2/x^2$ on the interval $[1, c]$ is equal to 1. Find the value of c.

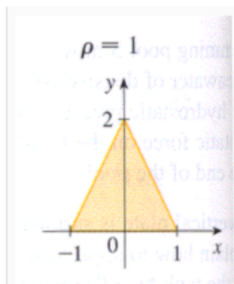
SOLUTION

The average value of $v(x)$ on the interval $1 \leq x \leq c$ is

$$\frac{1}{c-1}\int_1^c \frac{2}{x^2}\,dx = \frac{1}{c-1}\left(-\frac{2}{x}\right)\Big|_1^c = \frac{1}{c-1}\left(\frac{-2}{c}+2\right) = \frac{2}{c}.$$

Since the average is one, we have $\frac{1}{c-1}\int_1^c \frac{2}{x^2}dx = 1$, and thus $\frac{2}{c} = 1$, so $c = 2$.

Solution 7:



By symmetry, the moment about y-axis, M_y, is 0. The moment about the x-axis is,

$$\begin{aligned}
M_x &= \int_{-1}^{1} \rho x \text{ Top function } dx \\
&= \int_{-1}^{0} x(2+2x)\;dx + \int_{0}^{1} x(2-2x)\;dx \\
&= \left[x^2 + \frac{2x^3}{3}\right]_{-1}^{0} + \left[x^2 + \frac{2x^3}{3}\right]_{0}^{1} \\
&= -\left(1 - \frac{2}{3}\right) + \left(1 + \frac{2}{3}\right) \\
&= \frac{4}{3}
\end{aligned}$$

Since the mass of the enclosed is 2 (product of area and density), the center of mass is $(\bar{x}, \bar{y}) = (\frac{M_y}{\text{Mass}}, \frac{M_x}{\text{Mass}}) = (0, \frac{4}{6})$.