Quiz 4 solutions

Solution 2: The error in the nth numerical approximation of $\int_a^b f(x)\,dx$ using Trapezoid, Midpoint, and Simpson's rule are

$$\text{error in } n\text{th Trapezoid approximation, } E_T(n) = \frac{K(b-a)^3}{12n^2},$$
$$\text{error in } n\text{th Midpoint approximation, } E_M(n) = \frac{K(b-a)^3}{24n^2}, \text{ and}$$
$$\text{error in } n\text{th Simpson's approximation, } E_S(n) = \frac{L(b-a)^5}{180n^4}$$

respectively. Notice that the error using Midpoint rule decreases slightly faster than Trapezoid rule because of the constants $\frac{K(b-a)^3}{24n^2} < \frac{K(b-a)^3}{12n^2}$ for all n. The error using Simpson's rule decrease fastest because $\frac{L(b-a)^5}{180n^4} < \frac{K(b-a)^3}{12n^2}$ for large n. So, the correct answer is: **Approximation using midpoint rule is slightly better than trapezoid rule but Simpson's rule provides the best approximation.**

Solution 3:

False: Let $a_n = L$ and $c_n = \frac{1}{n} + L + 1$ for all n. Consider $b_n = \frac{\sin(n)}{4} + L + \frac{1}{2}$. The sequence $\{b_n\}$ oscillates taking values between L and $L+1$ and does not converge.

Solution 4:

True: If $(a_n)_{n=1}^{\infty}$ is an increasing sequence, then $a_n < a_{n+1}$ for all $n \in \mathbb{Z}^+$, and so

$$a_1 < a_2 < a_3 < a_4 ... a_n < a_{n+1} < ...$$

which shows that a_1 is a lower bound of the sequence $(a_n)_{n=1}^{\infty}$.

Solution 5:

a) Since the sequence $\left\{\frac{6n}{2n+3}\right\}$ converges to 3 and not to 0, we can say that $\sum_{n=1}^{\infty} \frac{6n}{2n+3}$ diverges to ∞. Also, it can be easily verified that the ratio test turns out to be inconclusive in this case.

b) To check the convergence of the sequence $\left\{\frac{6n}{2n+3}\right\}$, we need to take the limit going to infinity.

$$\therefore \lim_{n\to\infty} \frac{6n}{2n+3} = \lim_{n\to\infty} \frac{6}{2+3/n} = 3$$

The limit exists and hence the sequence converges to 3.

Solution 6:

a) The geometric series

$$\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n} = \sum_{n=0}^{\infty} \left(\frac{x+2}{2}\right)^n$$

converges for all x that satisfy $\left|\frac{x+2}{2}\right| < 1$. From this inequality, we get

$$-2 < x + 2 < 2 \implies x \in (-4, 0)$$

for which the series converges.

b) The given series is $1 + \frac{(x+2)}{2} + \frac{(x+2)^2}{2^2} + \frac{(x+2)^3}{2^3} + ...$

We know that the sum of the infinite series is $\frac{a}{1-r}$ where a is the first term and r is the common ratio which is < 1 i.e. the series converges.

$$\therefore \text{Sum} = \frac{1}{1 - \frac{(x+2)}{2}} = \frac{-2}{x}$$

which is the sum of the series for those values of x.