MATH 104: Week 4 Learning Goals

Implicit Differentiation. Inverse Trigonometric functions. Elasticity of demand. Exponential Growth. Mean Value Theorem. Related Rates

Learning Goals

This week we cover 6 sections of material. We will cover implicit differentiation, section 2.11, inverse trigometric functions, section 2.12, Mean value theorem, section 2.13 of the CLP Notes. You will also learn about *price elasticity of demand*, which is not covered in the text. While such elasticities can be a big topic, we will cover it in an introductory way, and there are notes posted online. We will introduce exponential growth, especially as applied to continuous compound interest. This material is in section 3.3 of the CLP Notes. There is a short set of notes on continuous compound interest posted online. Lastly, we will cover Related Rates, which is in section 3.2 of the Course Notes.

Readings

- **Readings:** In the CLP Notes: Chapter 2.11 on Implicit Differentiation, Chapter 3.3 on Exponential Growth (particularly 3.3.3 on Population growth), Chapter 2.13 on the Mean Value Theorem (MVT), Chapter 3.2 Related Rates. There are also extra notes posted for Elasticity of Demand and Continuous Compound Interest.
- **Problems:** We encourage you to do some of the problems in each section as you work through it to test your understanding of the material. Answers and solutions to the problems are provided in the text. If the material is new to you, start with the basic problems and work towards more difficult problems. Even doing a small number of problems while you work through the material in the text will help build your understanding.

Learning Goals

The specific learning goals for this week are that by the end of the week and review homework, students should be able to:

1. explain what we mean by *implicit differentiation* and identify situations where they will use it;
2. carry out computations involving implicit differentiation;
3. find equations of tangent lines to graphs of implicitly defined functions;
4. find equations of normal lines to graphs of implicitly defined functions;
5. use the implicit differentiation to demonstrate the power rule for rational exponents;
6. use the logarithmic derivative of a function $f(x)$ to compute relative rates of change of $f(x)$ per unit change of x;
7. compute the *price elasticity of demand* and use it to determine the direction revenue changes when there is a change in price;
8. solve problems involving price elasticity of demand;
9. compare and contrast linear growth and exponential growth;

10. solve problems involving continuously compounded interest

11. state the Mean Value Theorem (MVT), recognize when the hypotheses of the MVT are satisfied, and draw logical conclusions based on it;

12. set up and solve related rates problems;

    By this, we mean, for example, given a draining tank, falling ladder or moving ship problem, or provided a model of another situation, students should be able to:

    (a) identify all the variables involved, make appropriate choices when a variable takes on constant values, and describe how they relate (using equations if relevant and/or writing a short paragraph);

    (b) draw a picture of the situation if needed;

    (c) interpret rates in terms of derivatives with the appropriate variables; and

    (d) derive an equation which describes how the relevant rates are related and solve in that equation for the desired target rate.

Some Food for Thought as You Study This Week

1. You should work to understand what a relative rate of change is. This concept is used heavily in economics. It is useful to think of relative rates of change in terms of percentage change.

2. We will focus on the *price elasticity of demand* in MATH 104. The key for mastering this material is to link it to the idea that elasticity of demand encodes information about $R'(p)$, the marginal revenue function as a function of price, and so tells you something about how revenue changes as you (or the markets) adjust price. The notes I've posted start with this idea. (This won't necessarily be your main starting point in economics, but MATH 104 is not an economics course.) Remember, the law of demand implies that if $q = f(p)$, then $f'(p) < 0$ for all p – that is, the higher the price, the lower the demand. This accounts for the way we think about the sign in the formula for elasticity of demand ϵ). Be sure to spend some time to make sure you understand these relationships.

3. Explain why $\epsilon = 1$ gives the maximum revenue. We will study more about optimization (finding maxima and minima) in the near future.

4. If you look at old MATH 104 exams posted on the departmental website, be sure to note that some of these past exams tended to use a definition of elasticity that meant it was a positive quantity. We are allowing elasticity to be negative.

5. You will study the basic exponential growth (and decay) model, especially as applied to continuously compounding interest. This material is found in section 3.3 of the CLP Notes and a short note on continuous compound interest. We won't spend much (if any) time on carbon dating and Newton's law of cooling, but the material on population growth is useful for understanding unlimited and limited growth.

6. You will see both exponential growth and decay in action through the concepts of *present value* and *future value*. You should work to become comfortable manipulating $A = Pe^{rt}$ to $P = Ae^{-rt}$, where P is the present value and A is the future value of something under the condition of continuously compounded interest.

7. Our treatment of the Mean Value Theorem will be basic, and you may wish to start with Rolle's Theorem, which you may find easier to visualize. The Mean Value Theorem is central to calculus, but we will restrict ourselves to some basic applications of it that may help you understand it. These are best tackled through the problems.

8. Related rates problems include some common types of problems such as ladders sliding down walls, balloons blowing up, and boats moving away from docks. It is certainly possible to build a repertoire of basic problems and their solutions, and this will likely help you build your expertise in solving such problems. However, be careful that you do not fall into the trap of thinking that all related rates problems must be drawn from these basic types.

9. We will posts some extra business type related rates problems for you to try.