MATH 104: Week 6 Learning Goals
Taylor Polynomial. Approximiation.

Learning Goals

Over the week of term, we will cover Approximation (Linear, Quadratic and the more general Taylor Polynomial). The main emphasis will be on linear approximation and quadratic approximation. This material is covered in section 3.4 of the CLP Notes.

Lectures, Readings, Assignments, and Workshops

- **Readings:** In the CLP Notes: Chapter 3.4.
- **Problems:** We encourage you to do some of the problems in each section as you work through it to test your understanding of the material. Answers and solutions to the problems are provided in the text. If the material is new to you, start with the basic problems and work towards more difficult problems. Even doing a small number of problems while you work through the material in the text will help build your understanding.

Detailed Learning Goals

The specific learning goals for this week are that by the end of week students should be able to:

1. explain *linear approximation* (also known as *tangent line approximation* and the *linearization* of a function) using a series of figures to describe the process of approximating the value of a function using linear approximation; this includes being able to relate the formula for linearization to the elements of such a picture (for example: what is the role of a, what is the role of x, where is the appropriate tangent line on the graph, where does the linear approximation appear in the picture);
2. use linear approximations to estimate the values of functions near a given $x = a$;
3. use linear approximation to approximate changes in the dependent variable given changes in the independent variable;
4. given the exact value, discuss the discrepancy with the linear approximation in terms of the second derivative (for example, whether it is an underestimate or overestimate);
5. analyze the worst-case error for a linear approximation of a function using a formula based on the second derivative of the function (section 3.4.9);
6. *NOTE:* We are not covering the material in section 3.4.7;
7. use the quadratic approximation to estimate the values of functions;
8. explain the difference between the 2nd order term in quadratic approximation and the worst-case error term for linear approximation;
9. find the nth degree Taylor polynomial of a given function with a given centre $x = a$;
10. use a Taylor polynomial to approximate the values of functions; and
11. use the table of derivatives of inverse trigonometric functions in calculations of derivatives.

Food for Thought As You Study This Week

1. You will study Taylor polynomials, but we will focus almost entirely on the linear and quadratic approximations in MATH 104. We will ask you to do some worst-case error analysis for the linear approximation, but not to work with the remainder formula more generally, though you may be assured the analysis is similar should you ever need to do it.

2. It will be worthwhile developing your understanding of the linear approximation and a worst-case error analysis based on $M(x-a)^2/2$ where M is an upper bound on the absolute value of the second derivative on the interval defined by x and a. *Note that while we have the Mean Value Theorem, which comes into play in this error analysis, you will want to focus on building your understanding of this worst-case error analysis geometrically.* It is useful to start by thinking about the constant approximation first and developing a sense of the worst-case error based on your understanding of the derivative. You can then proceed to use what you know about what the second derivative says about concavity to see how you might do a worst-case error analysis for the linear approximation.

3. Once you have them comfortable with the quadratic approximation, it will not be too hard to use the more general Taylor polynomials. Be sure to do plenty of examples here, and include our old friends e^x, $\cos x$, and $\sin x$.

4. For inverse trig functions, our goal is that you be able to use a table of derivatives with inverse trig functions to compute derivatives. Those taking Math 101, 103. or 105 next term will need this for doing inverse trig substitutions.