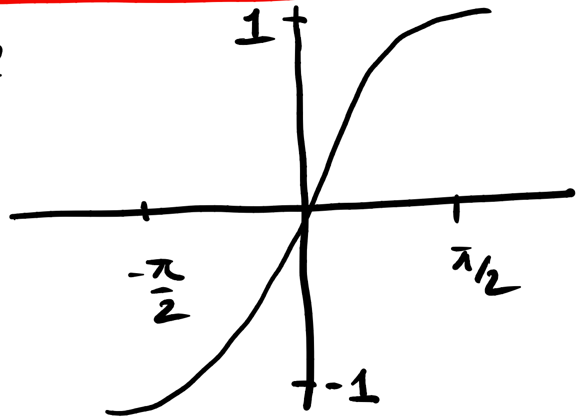
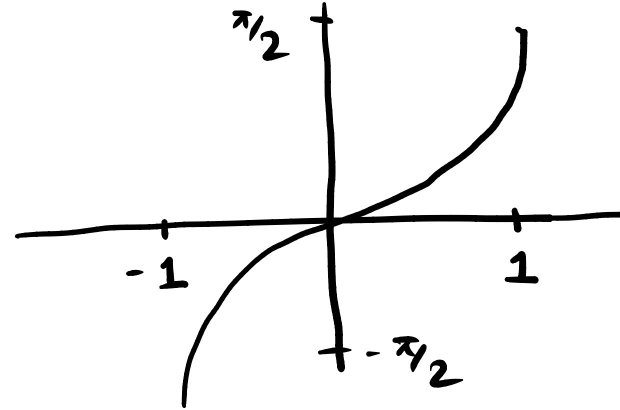
Inverse trig functions

Recall



sin(x)



arcsin(x)

For any $-1 \le y \le 1$, there exists one x such that

$$\sin(x) = y \quad \text{and} \quad -\pi/2 \le x \le \pi/2$$

The unique x is denoted as $\underline{\arcsin(y)}$.

$$\sin(\arcsin(y)) = y \quad \text{and} \quad -\pi/2 \le \arcsin(y) \le \pi/2$$

Let $\theta(x) = \underline{\arcsin(x)} \rightarrow$ $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is the angle such that $\sin(\theta) = x$

Use implicit differentiation:

Note that $\sin(\theta(x)) = x$

$$\frac{d}{dx}\left(\sin(\theta(x))\right) = 1$$

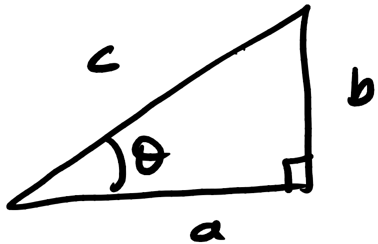
$$\Rightarrow \cos(\theta(x)) \cdot \theta'(x) = 1$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{1}{\cos(\theta(x))} = \frac{1}{\cos(\arcsin(x))}$$

we can simplify this more!

What is $\cos(\arcsin(x))$?

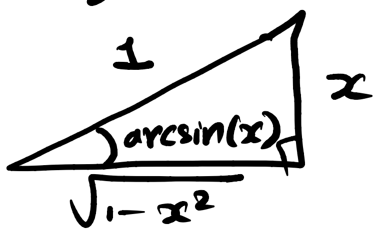
Note:



$\Rightarrow$ $c^2 = a^2 + b^2$

, $\sin\theta = \frac{b}{c}$, $\arcsin\left(\frac{b}{c}\right) = \theta$

Draw the right angled triangle for $\arcsin\left(\frac{x}{1}\right)$?



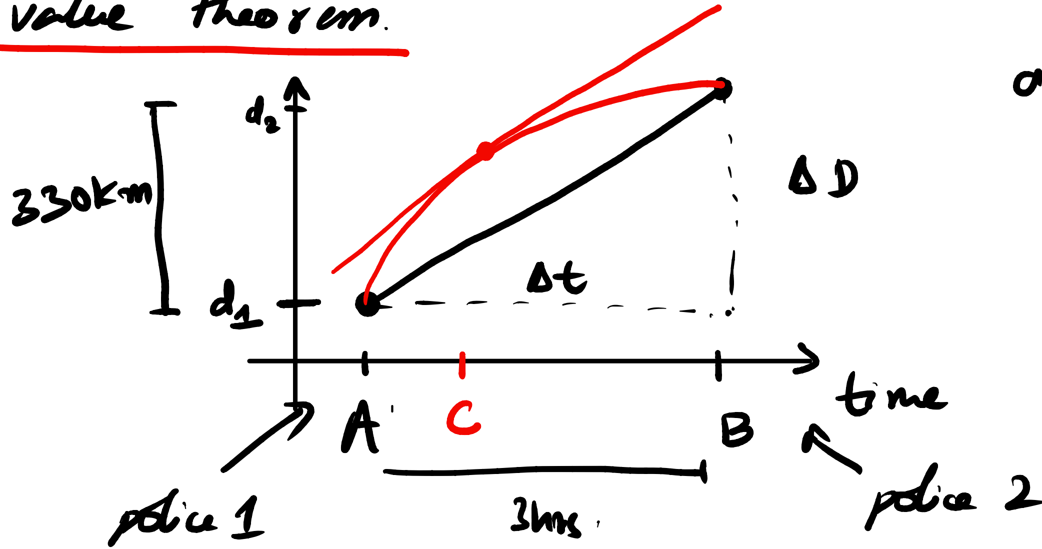
$\Rightarrow$ $\cos(\arcsin(x)) = \frac{\sqrt{1-x^2}}{1}$

Find derivations of other inverse trigs.

So, $\frac{d}{dx}\arcsin(x) = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}}$.

$f(x)$ we know $f'(x)$. suppose $g(x)$ is the inverse of f.

Mean value theorem.



average speed is $\frac{\Delta D}{\Delta t}$

After a phone call between police 1 and police 2, police 2 fines the driver for going 110 km/hr at some point between A & B. Why?

Average speed = 110 km/hr $\Rightarrow$ At some point between A & B, instantaneous speed was 110 km/hr.

Mean value theorem

Thm Let $a < b$ be real numbers. Let $f(x)$ be a function so that:

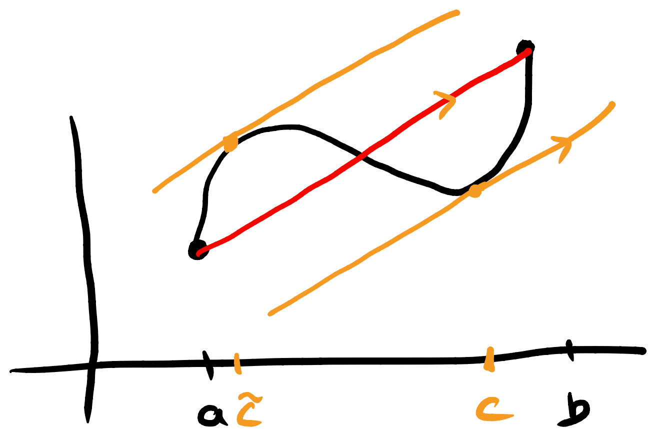
- $f(x)$ is continuous on the closed interval $a \le x \le b$
- $f(x)$ is differentiable on the open interval $a < x < b$

Then, there exists a point $c \in (a,b)$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b-a}.$$

slope of tangent line at c

slope of the secant line.



Example

Consider $f(x) = \underline{3x^2 - 4x + 2}$ on $[-1, 1]$.

Does MVT apply to $f(x)$ on $[-1, 1]$.

- $f(x)$ is a polynomial, hence its continuous and differentiable in the interval. So, MVT applies.

Q:

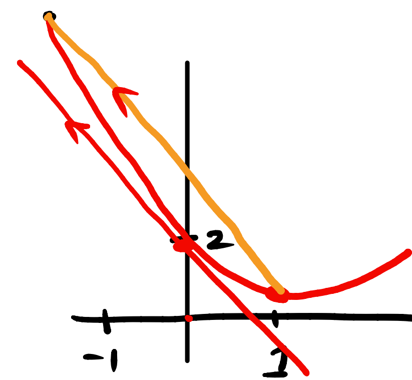
Find all values c in $[-1, 1]$ guaranteed by MVT.

Let c be a point where

$$f'(c) = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{1 - 9}{2} = -4$$

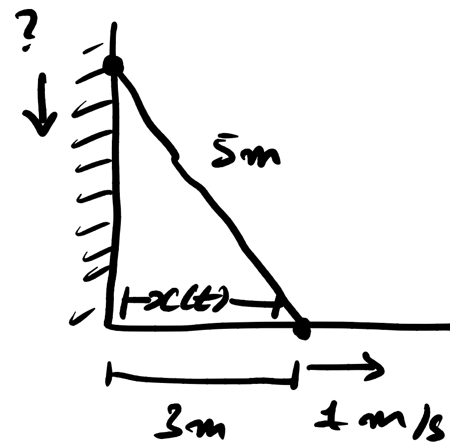
and $f'(x) = 6x - 4$

Hence $f'(c) = 6c - 4 = -4 \Rightarrow c = 0$



Related rates:

Q: A 5m tall ladder is leaning againts a wall. The floor is slippry and the base of ladder slides out from wall at a rate of 1m/s. How fast is the top of the ladder sliding down when the base of the ladder is 3 m from the wall



Sol^n:

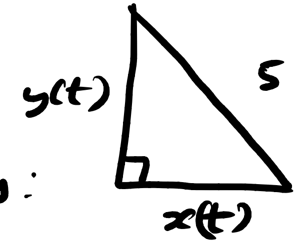
$x(t)$ is the distance of ladder to the wall at time t

$y(t)$ is the distance of top of ladder to the ground at time t.

Given: $x'(t) = 1$ m/s. Find $y'(t)$ when $x(t) = 3$m.

rate of change

we know : $\boxed{x(t)^2 + y(t)^2 = 5^2}$



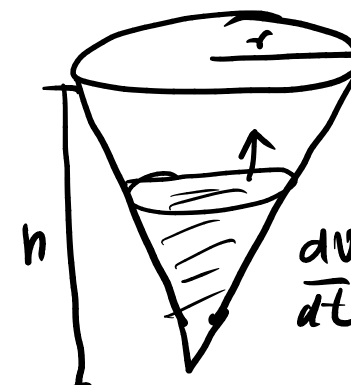
to find $y'(t)$, differentiate both sides :

$$\frac{d}{dt}x(t)^2 + \frac{d}{dt}y(t)^2 = 0$$

$$\Rightarrow 2\underbrace{x(t)}_{3m}\cdot\underbrace{x'(t)}_{1m/s} + 2\underbrace{y(t)}_{?}\cdot\underbrace{y'(t)}_{?} = 0$$

Find $y(t)$ when $x(t) = 3$:

$$9 + y(t)^2 = 25 \Rightarrow y(t) = 4$$



Lastly : $6 + 2 \cdot 4\, y'(t) = 0$

$$\Rightarrow y'(t) = -6/8 = -3/4 \text{ m/s}.$$

$$V = \frac{1}{3}\pi r^2 h$$