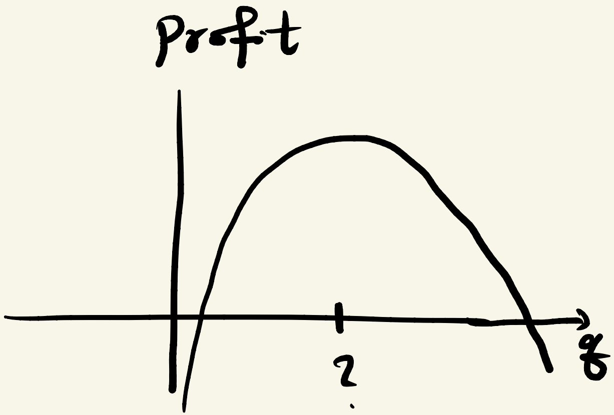
Review of chapter 0 and Appendix A

Simple setting:

In a bussiness setting, maximize profit from selling q # of a product at per unit price p in the presence of cost (fixed cost and variable cost).

$$\text{Profit} = \underset{?}{\text{Revenue}} - \underset{?}{\text{Cost}}$$

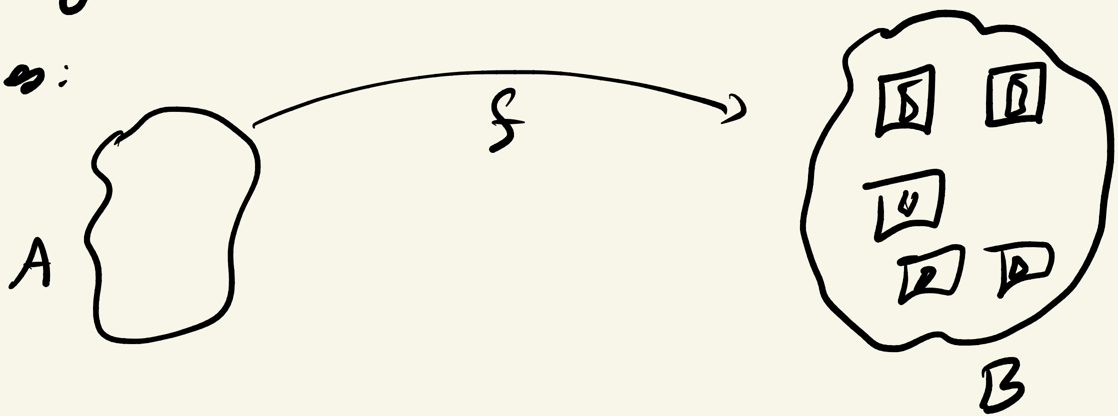
How do we maximize profit.



A complex setting:

2) Image deblurring: How do we remove noise, blurs, or other artifacts from an input image?

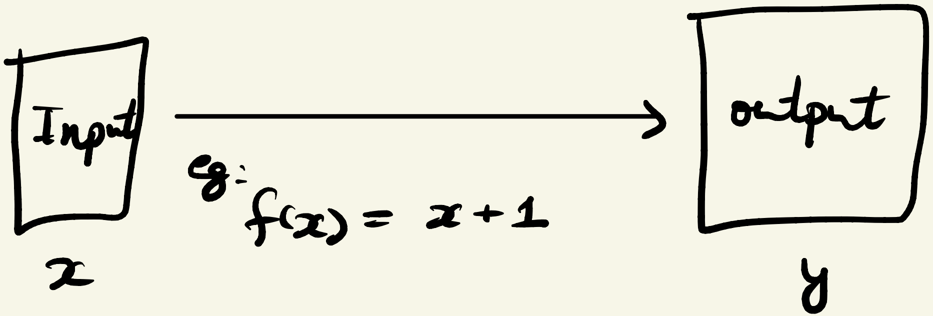
<u>One way</u>: If we have a way to generate clean images:



Find the image in the set B that is "closest" to the input blurry image

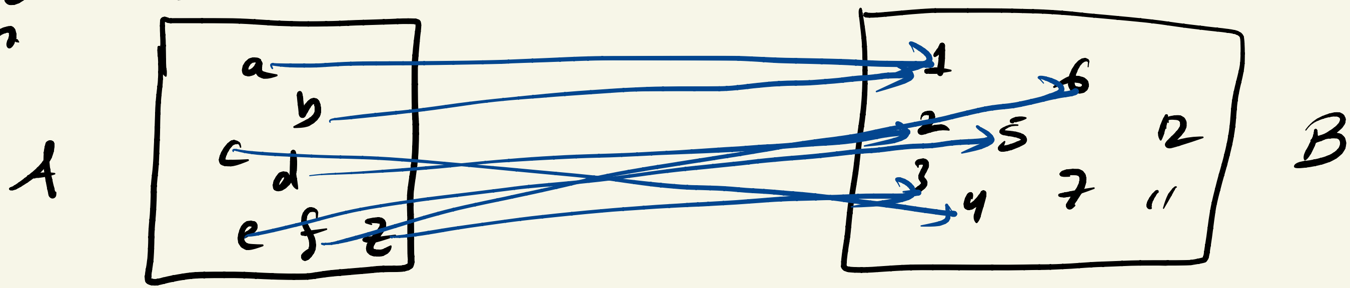
Functions

Functions as a formula:



eg: $f(x) = x+1$

More generally, a formula is not needed to define a function



"f maps a to 1", "f of a is 1"

Functions

$B = \{1, 2, 3, 4\}$ $\quad 5 \notin B$

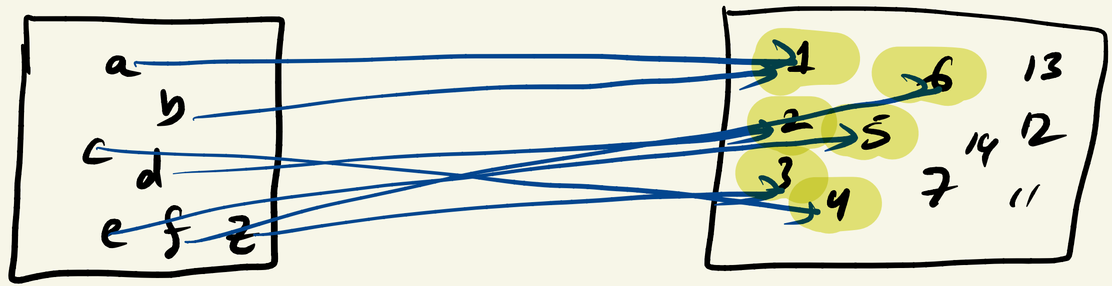
$1 \in B$ "1 belongs to B"

Let $f : A \to B$

1. The set A is the domain of f.
2. The set B is the codomain of f (contains outputs of f)
3. The range of f is

$$\text{range}(f) = \{ b \in B \mid \text{there exists some } a \in A \text{ so that } f(a) = b \}$$

A



B

domain

codomain.

$\subset$

$\subseteq$

$\text{Range}(f) = \{1, 2, 3, 4, 5, 6\} = \{1, 2, \ldots, 6\} \subseteq B.$

↑ contains.

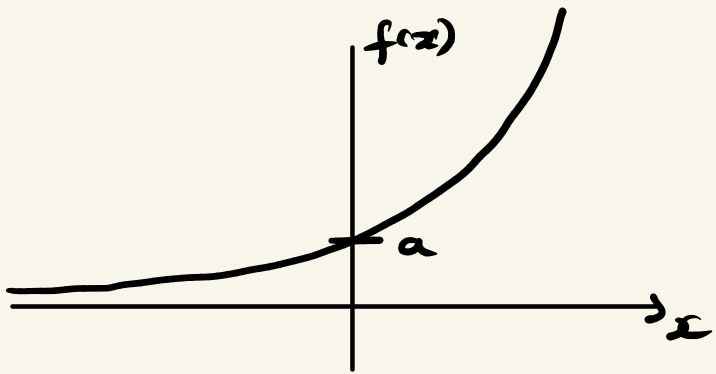
Exponential Function

2^x, 10^x, e^x, etc are all exponential functions.

An exponential function is a function of the form

$f(x) = ab^x$, where $a \in \mathbb{R}$, $b \in \mathbb{R}$

set of real numbers.



exponential functions capture exponential growth

$ab^{rx} \rightarrow$ more general

Most important case is $y = e^x$

$e = 2.718\ldots$ Euler's constant.

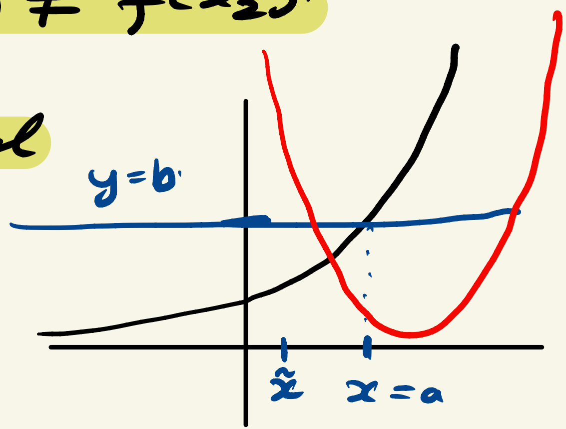
One-to-one function

Exponential function $f(x) = ab^x$ is one-to-one.

Def^n: A function f is one-to-one (also called injective) when two unique elements in domain of f does not map to the same element in the range. That is:

$$\text{if} \quad x_1 \neq x_2, \text{ then } \quad f(x_1) \neq f(x_2).$$

Def^n: A function passes the horizontal line test if no horizontal line intersects the graph $y = f(x)$ more than once



only a maps to b.

Logarithmic functions

The logarithmic function of base q $f(x) = \log_q(x)$ outputs a number that q must be raised to to give x.

"what power of q gives x"

Logarithmic functions is defined for any $q > 0$ except for $q = 1$. We only consider $q > 1$. eg: $q = e, 2, 10$

Defn: Let $q > 1$. Then the logarithmic with the base q is defined by

$$y = \log_q(x) \iff x = q^y$$

if and only if

$\log_{10}(100) = 2$

Logarithmic functions.

Note that

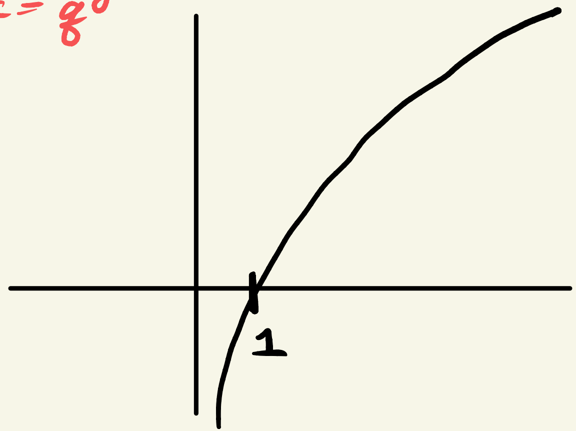
$$\log_q(q^x) = x \quad \text{and} \qquad \boxed{q^{\log_q(x)} = x}$$

why ?

because the power to which we have to raise q to get q^x is x.

$y = \log_q(x) \iff x = q^y$

$f(x) = \log_q(x)$ is a 1-1 function.



so, assume $q^{\log_q(x)} = \tilde{x} \equiv x$

by definition $\log_q(x) = \log_q(\tilde{x})$

$\Rightarrow x = \tilde{x}$ because log is 1-1.