MATH 104 Workshop 1

Functions and Inverse Functions, A Business Problem, Average Rate of Change of a Function

May 14, 2021

1. Consider the function
$$f(x) = \frac{x+1}{2x+1}.$$
   (a) Find the domain and the range of f.
   (b) Show that f is one-to-one on its entire domain.
   (c) Find f^{-1}, the inverse function of f.

2. M-Wave Co. makes and sells the world's first pocket quantum computers! When each quantum computer is sold for \$500, the weekly demand is 4,000 units. For every \$1 increase in the price of each unit, the number of quantum computers sold per week decreases by 10. Assume that it costs \$300 to produce each quantum computer.
   (a) Find the linear demand equation for the M-Wave quantum computer. Use p for the unit price and q for the weekly demand.
   (b) Find the weekly cost function $C(q)$ *as a function of* q.
   (c) Find the weekly revenue function $R(q)$ *as a function of* q.
   (d) Find the weekly profit function $P(q)$ *as a function of* q.
   (e) Sketch the Cost, Revenue, and Profit functions on the same set of axes, with q as the horizontal axis.
   (f) Find the break-even points for the M-Wave quantum computer. Give both the price p and quantity q at each of these points.
   (g) Suppose that M-Wave is producing and selling $\hat{q}$ quantum computers, where $\hat{q}$ corresponds to the largest q-value of all the break-even points. Should M-Wave increase or decrease the price of its robots to increase its profit? Explain your answer.

3. Consider the function $f(x) = x^2 + 2$.
   (a) Find the average rate of change of $f(x)$ with respect to x over the interval $[2, 5]$.

(b) Find the average rate of change of $f(x)$ with respect to x over the interval $[2, a]$ for some $a > 2$. Now take the limit of this average rate of change as a approaches 2. Explain how you can consider this to be the instantaneous rate of change of $f(x)$ with respect to x at $x = 2$ and give a geometric interpretation of the result.

Work Product for Workshop 1 (This applies to you only if you did not attend the workshop):

After this workshop, you will have 3 days to submit a *formal* solution to Problem 2 of this workshop.