MATH 104 Workshop 2
Continuity and Derivative

Work Product for Workshop 2 (This applies to you only if you did not attend the workshop): After this workshop, you have 3 days to submit a *formal* solution to **Problem 2 and 4** of this workshop.
Due date: 05/24, 5 pm

1. Consider the function

$$f(x) = \begin{cases} x & \text{if } 0 \le x < 1, \\ x - 1 & \text{if } 1 \le x \le 2, \\ 3 - x & \text{if } 2 < x < 3, \\ 1 & \text{if } x = 3, \\ x - 3 & \text{if } 3 < x < 4. \end{cases}$$

   (a) Find the following limits or explain carefully why they do not exist: (i) $\lim_{x\to 1} f(x)$, (ii) $\lim_{x\to 2} f(x)$, and (iii) $\lim_{x\to 3} f(x)$.

   (b) Describe any discontinuities $f(x)$ has on the interval $[0, 4]$.

2. If a function f is not defined at $x = a$ but the limit $\lim\limits_{x\to a} f(x) = L$ exists, then we can define a *continuous extension* of f to $x = a$ as the function F given by

$$F(x) = \begin{cases} f(x) & \text{if } x \text{ is in the domain of } f, \\ L & \text{if } x = a. \end{cases}$$

   Consider the function $f(x) = \dfrac{1 + x^3}{1 - x^2}$, which is not defined at $x = -1$. Find a function $F(x)$ that is a continuous extension of $f(x)$ to $x = -1$. What is the domain of $F(x)$?

3. Use the Intermediate Value Theorem (IVT) to show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$.

4. Carefully state the definition of the derivative of a function $f(x)$ at a point $x = a$. Use this definition to compute $f'(2)$ for $f(x) = \sqrt{4x + 1}$. (Do not use the rules of differentiation to do this calculation.)