

**MATH 104 Workshop 7**  
Optimisation Problems

**If you did not attend the workshop or were asked to submit solutions to the workshop problem via email, please submit solutions to problems 2, 4, and 5**

1. Two positive numbers sum to 7. What is the largest possible value for their product?
2. (a) Among all rectangles of a given area, show the square has the least perimeter.  
(b) Among all the rectangles of a given perimeter, show the square has the largest area.
3. A box is to be made from a rectangular sheet of metal 70 cm by 150 cm by cutting equal squares out of the four corners and bending the resulting flaps to make the sides of the box. (The box has no top.) What is the largest possible volume of the box?
4. Find the shortest distance from the point  $(8, 1)$  to the curve  $y = 1 + x^{3/2}$ .
5. Find the dimensions of the right-circular cylinder of greatest volume that can be inscribed in a sphere of radius  $R$ .
6. You are on a motorcycle in the desert 12 km due south of the nearest point  $A$  on a straight east-west road. You wish to get to a point  $B$  on the road 10 km east of point  $A$ . If your motorcycle can average 15 km/h traveling over the desert and 39 km/h traveling on the road, toward what point on the road should you head in order to minimize your travel time to  $B$ ?